Renormalizable parameters of the sine-Gordon model

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Abstract

The well-known phase structure of the two-dimensional sine-Gordon model is reconstructed by means of its renormalization group flow, the study of the sensitivity of the dynamics on microscopic parameters. Such an analysis resolves the apparent contradiction between the phase structure and the triviality of the effective potential in either phases, provides a case where usual classification of operators based on the linearization of the scaling relation around a fixed point is not available and shows that the Maxwell-cut generates an unusually strong universality at long distances. Possible analogies with four-dimensional Yang-Mills theories are mentioned, too.

Key words: Renormalization group, sine-Gordon model
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Introduction The phase structure and renormalizability of field theoretical models are well understood in the context of the renormalization group: Phase transitions belong to the singularities of the effective IR coupling constants as the functions of the UV parameters and the renormalizable parameters are the relevant or marginal coupling constants of the UV fixed point. The two-dimensional sine-Gordon (SG) model has a well-known phase structure and renormalization group flow but does not easily fit into the general scheme. Our aim in this work is the clarification of these issues by a careful renormalization group study of the SG model.

The SG model has an ionized (massless, strong coupling, non-renormalizable) phase for $\beta^2 > 8\pi$ and a molecular (massive, weak coupling, renormalizable) phase for $\beta^2 < 8\pi$. The molecular phase is perturbatively equivalent with the neutral sector of the massive Thirring model [1, 2] and the neutral Coulomb-gas [3]. Perturbation expansion was used to conjecture the same universal behavior around $\beta^2 = 8\pi$ [4, 5] and phase structure [6] as that of the planar X-Y model. Simple comparison of the lattice regulated SG model with the planar X-Y model provides a non-perturbative renormalization group flow for the SG model [7]. The SG model, defined in the continuum by aperiodic kinetic energy, has no or few renormalizable coupling constants in the ionized and the molecular phases, respectively. It has been argued that the appropriate order parameter, provided by the soliton dynamics, is the topological susceptibility [8], because the symmetry with respect to the fundamental group $\pi_1(U(1))$, $\phi_x \to \phi_x + \Delta\phi$, $\Delta\phi = 2\pi/\beta$ being the period length of the local potential, is broken dynamically in the molecular phase.

But some complication arises by the similarity of the deep infrared IR dynamics in the two phases of the SG model [8]. The rather trivial observation, namely that the only non-singular effective potential which is periodic and convex is the constant shows that the Maxwell-construction washes away the differences of the phases in the deep IR regime. How can this be reconciled with the overwhelming evidences about the two differing phases and the different numbers of the renormalizable parameters? We show that the difference of the two phases appears in the effective potential expressed in units of the running cutoff. This potential produces weak effects in the deep IR but can formally distinguish the phases by developing singularities in the molecular phase. This difference can be used to identify the phase structure by studying the sensitivity of the dynamics at a given observational scale as the function of the bare parameters, given at the cutoff scale [9]. Such a global use of the renormalized trajectory is needed because of the non-triviality of the IR scaling laws, the inherent non-linear nature of the renormalization group trajectory at the IR fixed point of the ionized phase.

Blocking The low energy behavior of the SG model in Euclidean spacetime will be determined by means of the differential renormalization group (RG) method with gliding sharp cut-off k in momentum space [9, 10, 11, 12]. The infinitesimal blocking is followed by a blocked action corresponding to a particular functional subspace. In the local potential approximation (LPA) the blocked action is given as

$$S_k = \int_x \left[\frac{1}{2}(\partial_\mu \phi_x)^2 + U_k(\phi_x)\right], \tag{1}$$

where the local potential is represented by the Fourier series

$$U_k(\phi) = \sum_{n=1}^{\infty} u_n(k)\cos(n\beta\phi) \tag{2}$$

and satisfies the Wegner-Houghton equation [13]

$$(2 + k\partial_k)\,\tilde{U}_k(\phi) = -\frac{1}{4\pi}\ln\left(1 + \tilde{U}_k''(\phi)\right), \tag{3}$$

$\tilde{U}_k = k^{-2}U_k$ denoting the dimensionless local potential. The sharp cut-off is needed to handle spinodal instabilities and condensates when non-trivial saddle-point ϕ' appears in the blocking procedure [14, 15] leading to the tree-level blocking relation

$S_{k-\Delta k}[\phi] = \min_{\phi'}(S_k[\phi+\phi'])$. The search of the saddle point among plane waves $\phi'_x = \rho\exp(ikx)$ yields the evolution equation [8, 9]

$$\tilde{U}_{k-\Delta k}(\phi) = \min_{\rho}\left[\rho^2 + \frac{1}{2}\int_{-1}^{1} du\tilde{U}_k(\phi + 2\rho\cos(\pi u))\right] \tag{4}$$

replacing Eq. (3) in the unstable region. It is easy to show that the RG flow in LPA preserves the period length of the potential. The evolution equation will be integrated from the initial condition imposed at the cutoff $k = \Lambda$.

The solution of the RG equation in the UV scaling regime, $k^2 \gg |U''_\Lambda|$, is

$$\tilde{u}_n(k) = \tilde{u}_n(\Lambda)\left(\frac{k}{\Lambda}\right)^{n^2\frac{\beta^2}{4\pi}-2} \tag{5}$$

after ignoring contributions $\mathcal{O}(k^2/|U''_\Lambda|^2)$, displaying the well-known critical point at $\beta^2 = 8\pi$. The relevant coupling constants of the UV fixed point are called renormalizable because those parameterize the dynamics at finite scales when the cutoff is removed. The two-dimensional scalar model with polynomial interactions possesses infinitely many renormalizable operators. In contrast, the periodicity of the potential of the SG model seems to make all parameters of the local potential UV irrelevant, i.e. non-renormalizable in the ionized phase, $\beta^2 > 8\pi$ and to allow only a few renormalizable parameters in the molecular phase, $\beta^2 < 8\pi$. But one has to consider the renormalized trajectory globally, by taking into account the IR scaling laws in order to find the free parameters of the renormalized dynamics. In fact, the number of these parameters might be more or less if the IR scaling has new relevant operators or stronger universal features, respectively [9]. We now present a global study of the renormalization group flow to point out that the local analysis at a given fixed point is actually not reliable in the SG model.

The solution of the RG equation can only be obtained numerically in the IR region because of the mixing of the different Fourier modes and the appearance of condensate for $\beta^2 < 8\pi$ [8]. It was found that the dimensionful potential is vanishing at low energy, $U_k(\phi) \to 0$ as $k \to 0$ in either phase. But the dimensionless potential has different shape in the two phases.

Ionized phase There is no spinodal instability and the renormalized trajectory can be well approximated in the IR scaling regime by the simple power law $\tilde{u}_n = c_n(k/k_0)^{n\eta}$, with $\eta \geq 0$, k_0 being some scale parameter. In fact, the evolution equation reads as

$$(2+n\eta)nc_nk^{n\eta} = \frac{\beta^2}{4\pi}n^3c_nk^{n\eta} + \frac{1}{2}\beta^2\sum_{s=1}^{\infty}sA_{n,s}(2+s\eta)c_sk^{s\eta} \tag{6}$$

when this assumption is made. For $n = 1$ one finds $\eta = \beta^2/4\pi - 2 > 0$, $c_1 =$

$\tilde{u}_1(\Lambda)(k_0/\Lambda)^\eta$ and the cases $n > 1$ lead to the recursion relation

$$c_n = \frac{\frac{1}{2}\beta^2 \sum_{s=1}^{n-1}(2+s\eta)s(n-s)^2 c_{n-s}c_s}{n(2+n\eta-n^2\frac{\beta^2}{4\pi})}, \tag{7}$$

expressing c_n in terms of c_1, $c_n = (-1)^{n+1}\tilde{u}_1^n(\Lambda)R_n$, where $R_1 = 1$ and the R_n satisfying the recursion relation (7) becomes independent of the bare couplings. This scaling law is confirmed by the numerical results, shown in Fig. 1 for $\eta = 1$, i.e. $\beta^2 = 12\pi$. Further numerical support is that the ratio $R_n = |\tilde{u}_n(k)|/\tilde{u}_1^n(k) = |c_n|/(c_1)^n$ becomes k-independent and the local potential becomes independent of $u_n(\Lambda)$, $n > 1$ in the IR region.

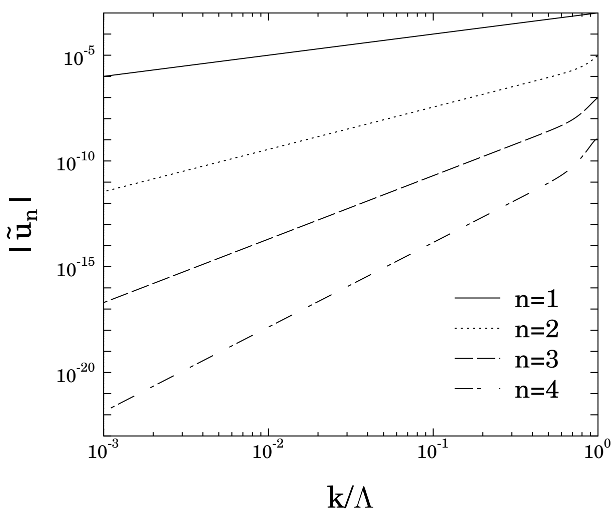

Fig. 1. The IR scaling law at $\beta^2 = 12\pi$ for various Fourier amplitudes.

A remarkable complication takes place at the IR fixed point, $\tilde{u}_n^\star = 0$. The upper harmonics with $n > 1$ decrease too fast as $k \to 0$, $\mathcal{O}(\tilde{u}_n) = \mathcal{O}(\tilde{u}_1^n)$, and the renormalizable trajectory is not linearizable around the fixed point. The problem is that one tacitly assumes in the usual argument in linearizing the blocking relation $\tilde{u}'_n = B_n(\tilde{u})$,

$$\tilde{u}'_n - \tilde{u}^\star_n = (\tilde{u}'_m - \tilde{u}^\star_m)\partial_m B_n(\tilde{u}^\star) + \mathcal{O}((\tilde{u}'_m - \tilde{u}^\star_m)^2), \tag{8}$$

that the deviation of each coupling constant from the fixed point values is of the same order of magnitude. As a result, we do not have the usual classification of operators and a more complete study, based on the global features of the renormalization group trajectory, is needed to determine the number of free parameters of the renormalized dynamics. The results like the one shown in Fig. 1 indicate that scaling laws characterized by critical exponents are actually recovered in a non-trivial, non-linear manner with no relevant parameter. This result is in agreement with the picture suggested by the analogy with the X-Y model [7] where the vortex fugacity is found to be the only relevant parameter in this phase. But these configurations have divergent action for the SG model defined in terms of non-periodic space-time derivatives. In other words, our results correspond to the vanishing fugacity hyperplane in the space of coupling constants.

Molecular phase The renormalized trajectory follows scaling laws similar to those of the ionized phase at the beginning. Namely, the asymptotic scaling law (5) in the UV regime ends at a crossover beyond which the scale dependence $\tilde{u}_n \sim k^{n\eta}$ is encountered. The crucial difference between the two phases appears in this region in what this scaling law does not extend to $k = 0$ as in the ionized phase, rather it is interrupted by a further crossover. The period length of the potential is larger than in the ionized phase and as a result the symmetry $\phi_x \to \phi_x + 2\pi/\beta$, belonging to the fundamental group of the theory, is broken spontaneously at low energy where the potential has more chance to localize the field around a given minimum by the formation of inhomogeneous saddle-points to the blocking relation. The second crossover mentioned above, a spinodal instability appears when the propagator diverges, $k_{\rm SI}^2 + U''_{k_{\rm SI}}(\phi) = 0$. The blocked action can easily be determined when $\beta^2 \to 8\pi$ from below because $\tilde{u}_1$ is the only renormalizable coupling constant. Due to $k_{\rm SI}/\Lambda \ll 1$ the UV irrelevant couplings die out and it is sufficient to keep track of $\tilde{u}_1$ only, i.e. one can estimate the value of $k_{\rm SI}$ by calculating the potential $U_k(\phi)$ as if it would contain a single Fourier mode only. Using $\tilde{u}_1(k) = \tilde{u}_1(\Lambda)(k/\Lambda)^\eta$ we find

$$k_{\rm SI}^2 = \Lambda^2 \left(\beta^2 \tilde{u}_1(\Lambda)\right)^{\frac{8\pi}{8\pi-\beta^2}} . \tag{9}$$

We used here the fact that the minimum of $U''_k(\phi)$ lies at $\phi = 0$ in the case of a single coupling. The numerically determined running of the coupling constant $\tilde{u}_1(k)$ is depicted in Fig. 2. Note that the length scale $1/k_{\rm SI}$ where the spinodal instability occurs is not an analytic function of β at $\beta^2 = 8\pi$. This situation resembles to the non-analyticity of the correlation length at the Kosterlitz-Thouless phase transition point [6].

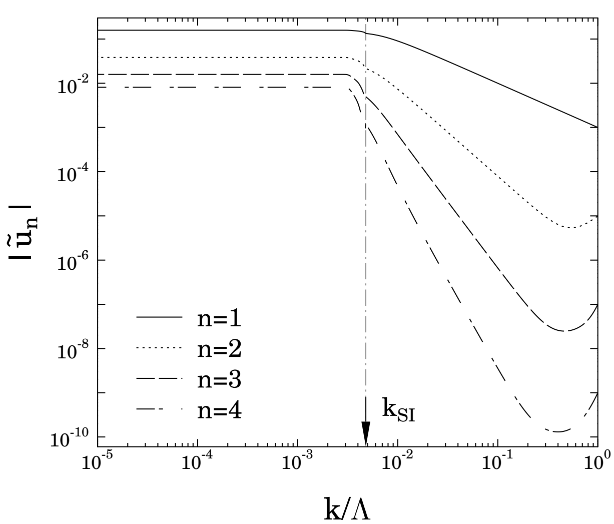

Fig. 2. Scale-dependence of the couplings $\tilde{u}_n$, with $n = 1 \dots 4$ at $\beta^2 = 4\pi$. The scale $k_{\rm SI}$ where the spinodal instability appears is shown explicitly.

Once the cutoff has reached the spinodal instability region its further decrease strengthens the condensate and flattens the potential in order to arrive at a constant potential for $k \to 0$, being the only function which is simultaneously convex and periodic [8]. The numerical results, displaying this phenomenon are presented in Fig. 3.

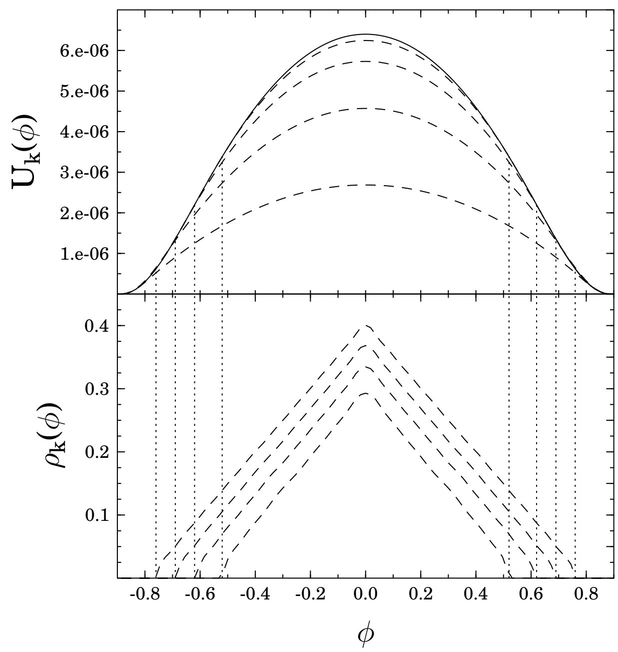

Fig. 3. The dimensionful potential and the amplitude of the condensate are plotted for decreasing values of the cut-off for $\beta^2 = 4\pi$ as the functions of the homogeneous background field. The potential shown by solid line corresponds to $k = k_{\mathrm{SI}} \approx 0.0046$ when the spinodal instability appears. The potential flattens out and the condensate grows as k is decreased.

Though the dimensionful effective potential is flat in both phases, the dimensionless effective potential $\tilde{U}_0(\phi)$ is not flat in the molecular phase where one has $\tilde{U}_0(\phi) = -\frac{1}{2}(\phi - n\Delta)^2$ for $(n - \frac{1}{2})\Delta \le \phi < (n + \frac{1}{2})\Delta$. Both the linear dependence of the strength of the saddle point on the background field and the quadratic shape of the dimensionless potential can be understood by simple analytical considerations [16].

Note that the spinodal instability and condensation also induce an effective potential of parabolic shape when the periodic bare potential is replaced by a polynomial one. The only difference is the lack of periodicity in the latter case. The universal parabolic shape is a direct consequence of the LPA and the Maxwell-cut which have to be performed on the unstable region of the effective potential [14, 15].

In order to make the approach of the potential to a parabola more apparent we numerically determined $\beta^2 \tilde{u}_n(0)$ and found that it is a universal function of n, $\beta^2 \tilde{u}_n(0) = 2(-1)^n/n^2$ for $\tilde{u}_1(\Lambda) > 0$, cf the inset of Fig. 4. The corresponding

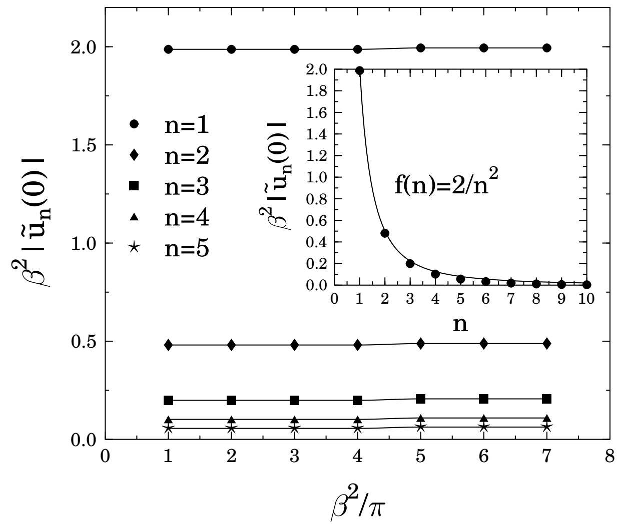

Fig. 4. The magnitudes of the first few dimensionless couplings are plotted as the function of β. The inset shows that the β-dependence can be easily factorized and the index-dependence also obtained.

dimensionless effective potential is

$$\tilde{U}_{k\to 0}(\phi) = \frac{2}{\beta^2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos(n\beta\phi)}{n^2} = -\frac{1}{2}\phi^2, \quad \phi \in [-\pi/\beta, \pi/\beta] \tag{10}$$

apart from a field independent constant. This parabola is repeated periodically along the ϕ axis. It is apparent that such a periodic potential is the solution of Eq. (4).

How is the parabolic shape formed as we approach the IR end point? The exactly parabolic shape part of the dimensionless blocked potential appears at $k = k_{\rm SI}$ only and spreads over larger field region as the cutoff is further lowered. But the potential approaches a parabola-looking shape already before the appearance of the condensate as demonstrated in Fig. 5, where the difference $\Delta\tilde{U}_k(\phi_n) = \tilde{U}_k(\phi_n) - (-\frac{1}{2}\phi_n^2)$ is plotted at $\phi_n = n\pi/(10\beta)$ with $n = 0, 1, \ldots, 10$. The gradual approach of the potential to a parabolic shape is a precursor of the condensation. According to the numerical results $\Delta\tilde{U}_k(\phi_n)$ follows a power law behavior in a rather small region above $k_{\rm SI}$. The exponent characterizing this power law depends on β, the value $\nu = 3.75$ was found at $\beta^2 = 4\pi$ with ν decreasing as β is increased. Once the parabolic shape is approximately installed for k slightly above $k_{\rm SI}$ the potential does not suffer a sudden change with the appearance of the condensate and it assumes its parabolic shape very rapidly below $k_{\rm SI}$ in the whole period length.

The insertion of an aperiodic parabola section into a periodic potential and the joining of functions with different analytical structures are rather involved issues. The deviation of the dimensionless potential from the parabolic form around the matching points are found to be weak but not sufficiently fast converging in the Fourier expansion. The increase of the order of the truncation of the Fourier series

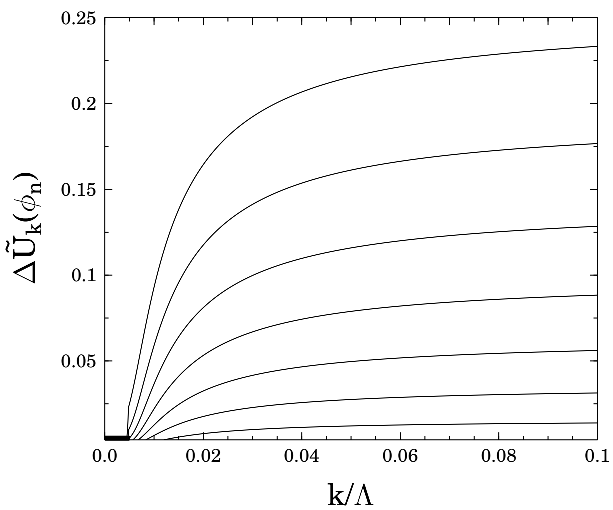

Fig. 5. The deviation of the dimensionless blocked potential from the parabolic shape at different values, $\phi_n = n\pi/(10\beta)$ with $n = 0, 1, \ldots, 10$, of the field variable. The deviation increases with increasing n.

makes the spread of the parabolic shape faster at $k \approx k_{\rm IS}$ but the second derivative of the potential increases at the matching points without bound, too. Unfortunately the WH method does not allow us to go beyond the LPA where deviations from the parabolic potential are compatible with the Maxwell-cut.

The dimensionless potential (10) displays a non-trivial structure in the molecular phase and is independent of the bare coupling constants, either relevant or irrelevant at the UV fixed point. Such a surprisingly strong super-universality renders the deep IR physics completely parameter-free. Though the UV relevant couplings influence the dynamics at finite scales, the physics becomes parameter-free well below the condensate scale.

The dimensionless local potential distinguishes the two phases of the SG model but this difference is vanishing in fixed, dimensionful units. Nevertheless the phases can still be distinguished by means of the renormalization group flow as shown in Fig. 6. It remains to be seen if this difference rests qualitatively valid when one goes beyond the LPA. In particular, it would be interesting to decide whether the couplings $\tilde{u}_n(k \to 0)$ remain finite [17] or diverge [4] beyond the LPA when the parameter β exhibits also scale-dependence.

Sensitivity matrix The sensitivity matrix is the systematical tool to distinguish phases of a theory according to the global features of the renormalization group flow. Let us consider the bare coupling constants, $\tilde{u}_n(k) = E_n(\tilde{u}(\Lambda), r)$, as a function of the ratio of the observational scale and the cutoff, $r = k/\Lambda$ and the bare parameters, identified by the initial condition $\tilde{u}_n(\Lambda) = E_n(\tilde{u}, 1)$. The sensitivity of the renormalization group flow at scale k to an infinitesimal change of the bare

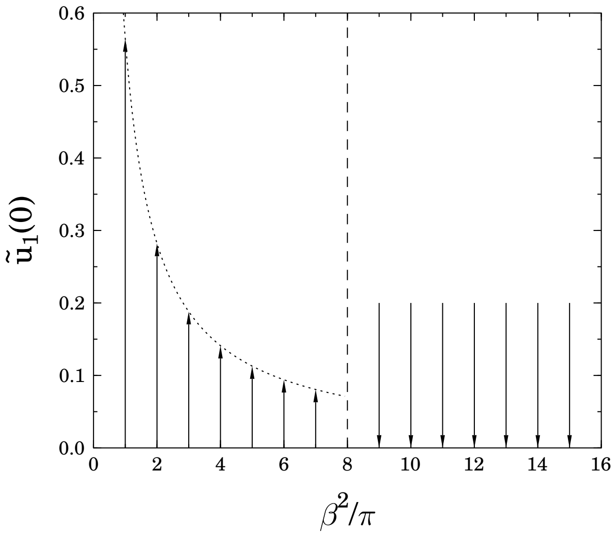

Fig. 6. The phase structure of the sine-Gordon model. The dotted line corresponds to $\tilde{u}_1(0) = \text{const.}/\beta^2$. The critical value $\beta^2 = 8\pi$ separates the two phases of the SG model.

couplings is characterized by the sensitivity matrix [9]

$$S_{n,m}(r) = \frac{\partial E_n(\tilde{u}(\Lambda), r)}{\partial \tilde{u}_m(\Lambda)}. \tag{11}$$

Phase transitions can be identified with the singularities in $S_{n,m}(r)$ as the function of the parameters of the theory in the limit $r \to 0$. The singularities developing as $r \to 0$ with fixed k and $\Lambda \to \infty$ correspond to quantum phase transitions because they are driven by short distance phenomena. The traditional phase transitions, induced by the low energy, long range modes belong to singularities realized with fixed Λ and $k \to 0$.

The UV scaling laws are given by Eq. (5),

$$S_{n,m}(r) = \delta_{n,m} r^{\frac{1}{4\pi}(n^2\beta^2 - 8\pi)} \tag{12}$$

in either phase. The situation changes significantly in the IR region, $r \ll 1$, where the scaling law

$$\tilde{u}_n(k) = (-1)^{n+1} R_n \tilde{u}_1^n(k) = (-1)^{n+1} R_n \tilde{u}_1^n(\Lambda) r^{n\frac{1}{4\pi}(\beta^2 - 8\pi)} \tag{13}$$

of the ionized phase yields

$$S_{n,m}(r) = \delta_{m,1} (-1)^{n+1} R_n n \tilde{u}_1^{n-1}(\Lambda) r^{n\frac{1}{4\pi}(\beta^2 - 8\pi)}. \tag{14}$$

The low energy dynamics looses sensitivity on any bare coupling constant when $\eta > 0$. This is supported by the numerical results for the sensitivity matrix, shown in Fig. 7 with dashed line. The lack of sensitivity is established in two different ways: both in the UV ($\Lambda \to \infty$) and the IR ($k \to 0$) limits. The renormalized SG model becomes trivial and has no free parameter in this phase even if the problem with the linearization of the blocking relation around the IR fixed point prevents us to classify the operators in the usual fashion.

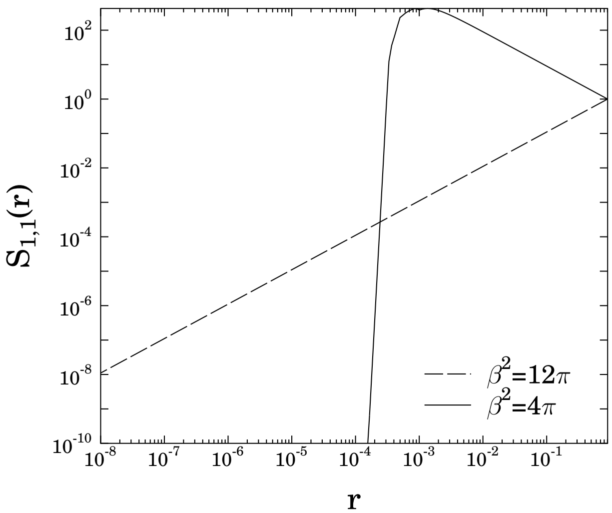

Fig. 7. The numerically calculated sensitivity matrix at $u_n(\Lambda) = 10^{-4}\delta_{n,1}$. The slope in the ionized phase is in an excellent agreement with $\eta = 1$.

When the limit $r \to 0$ is realized with fixed $k > k_{\rm SI}$ then the sensitivity matrix, given by Eq. (14) has a singularity at $\beta^2 = 8\pi$, in agreement with the UV origin of the phase transition in the SG model. The renormalized dynamics at such a scale k has indeed few free parameters, the relevant coupling constants of the UV scaling law. This phase transition is shown by the continuous line in Fig. 7 for $r > 10^{-3}$. But the approach of the potential to a parabolic shape as $k \approx k_{\rm SI}$ stops this trend. The dynamics depends less and less on the high energy, short-distance parameters as k is decreased below $k_{\rm SI}$ and the super-universality developing in this regime renders the dynamics free of any parameter as $k \to 0$.

The dimensionful potential shows the same, flat shape in both phases. Despite the clear difference of the dimensionless potential and the sensitivity matrix in the deep IR regime, indicating that the dynamics, observed in units of the low cutoff differs significantly in the two phases, the IR dynamics of these phases do not look different when expressed in fixed units. It is easy to understand this apparent contradiction. The two phases are similar in their IR plane-wave dynamics, both decouple them. But it is well-known that the SG model has other important degrees of freedom, solitons, due to the periodicity of the potential. Their finite size or mass introduces another important scale where the dynamics differ in the two phases.

Summary The phase structure and the number of free parameters of the SG model were studied in this work. The phase structure was identified without using topological disorder parameters, by means of the renormalization group flow. We had to rely on the global analysis of the renormalization group flow due to an unexpected problem with the linearization around the IR fixed point of the ionized phase. Furthermore, it has been clarified that the sensitivity of the dynamics on the renormalizable coupling constants is washed away and a super-universality is generated in the IR limit of the molecular phase by the Maxwell-cut.

These features of the SG model show possible interesting analogies with four-dimensional Yang-Mills theories. Both models have periodic variables. The periodicity of the gauge model, the fundamental group symmetry is supposed to play a key role in establishing the confining forces between charges. The renormalizable, asymptotically free coupling constant of the Yang-Mills theory and the SG model in the molecular phase increase with the observational length scale. Such an increase generates condensate and spinodal instabilities in the vacuum of both theories. The speed of sound is vanishing in the mixed phase with spinodal instability, indicating the emergence of color confinement, the absence of asymptotical charged plane-wave states [15]. The exact scattering matrix constructed in terms of soliton-anti soliton bound states of the SG model suggests that the asymptotic states of this two dimensional model are made up by these composite bound states. The SG model realizes a mechanism to remove any free parameter in the dynamics of the condensate, well below its energy scale, in a manner similar to the long range dynamics of the Yang-Mills vacuum.

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